



Sensitivity analysis in an inventory model of deteriorating items involving fuzzy with shortages

Raish Muhammad

Department of Mathematics, Modern Institute of Technology & Research Centre, Alwar, Rajasthan, India

Abstract

Sensitivity analysis is presented in an inventory model of deteriorating items involving fuzzy with shortages and infinite production rate. It is observed that the time of one cycle highly decreased by increasing holding cost, purchasing cost, shortage cost and hazardous rate; the inventory period highly decreased by increasing the holding cost, purchasing cost and hazardous rate but highly increased by increasing the shortage cost; the average total cost per unit time highly increased by increasing the holding cost, purchasing cost, shortage cost and hazardous rate.

Keywords: sensitivity, infinite production rate, deteriorating items, inventory model, fuzzy with shortages

1. Introduction

Inventory is defined as the stock of items to satisfy the future demands. Harris ^[1] developed the mathematical model to decide the number of products at ones. He also gave the concept of economic order quantity (EOQ). After him many mathematical models have been developed for controlling the inventory. In several exciting models, it is assumed that the products have infinite shelf time. But actually deterioration plays a vital role in inventory. Deterioration is defined as decay, spoilage, loss of utility of products etc. The process of deterioration is observed in volatile liquids, beverages, medicines, blood components, sweets, fruits and vegetables. There are many other products in the real world which deteriorate with a significant rate. So it should not be neglected in the decision process of production lot size.

In recent years, mathematical ideas have been used in different areas in real life problems, particularly for controlling inventory. When the items of the commodity are kept in stock as an inventory for fulfilling the future demand, there may be deterioration of items in the inventory system.

At the end of the storage period, deterioration is studied by Whitin ^[2] for the fashion goods. Ghare and Schrader ^[3] analyzed the problem of decaying inventories exponentially and developed an EOQ model with constant demand. Covert and Philip ^[4] extended Ghare and Schrader's model by considering a two parameter Weibull's distribution for variable rate of deterioration. Shah and Jaiswal ^[5] developed an order-level inventory model for a system with constant rate of deterioration. Aggarwal ^[6] modified Shah and Jaiswal's model in calculating the average inventory holding cost. Yang and Wee ^[7] developed an integrated multi-lot-size production inventory model for deteriorating item. Sharma, *et al.* ^[8] developed a deterministic production inventory model for deteriorating products with exponentially declining demand and shortages. Baten and Kamil ^[9] studied the inventory management systems with two-parameter exponential distributed hazardous items in which production and demand rates are constant. Manna and Chiang ^[10] presented economic production quantity models for deteriorating items with ramp type demand. Mishra and Raju ^[11] developed a deterministic inventory model for deteriorating items with on-hand inventory dependent, variable type demand rate. Baten and Kamil ^[12] gave optimal fuzzy control with application to discounted cost production inventory planning problem. Sharma, *et al.* ^[13] developed an inventory model for hazardous items of two-parameter exponential distribution with finite production rate. Sharma and Muhammad ^[14] analysed correlation between parameters and variables of EOQ models without shortages for hazardous items. Mishra and Singh ^[15] gave computational approach to an inventory model with ramp-type demand and linear deterioration. Mishra and Singh ^[16] studied partial backlogging EOQ model for queued customers with power demand and quadratic deterioration. Sharma and Muhammad ^[17] presented an EOQ model for hazardous items with uniform rate of demand. Dash, *et al.* ^[18] gave an inventory model for deteriorating items with exponential declining demand. Sharma and Muhammad ^[19] developed an EOQ model with instantaneous production for hazardous items of two-parameter exponential distribution. Kumar, *et al.* ^[20] gave fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. Sharmila and Uthayakumar ^[21] presented inventory model for deteriorating items involving fuzzy with shortages and exponential demand. Sharma and Muhammad ^[22] analyzed sensitivity of inventory model for deteriorating items with on-hand inventory dependent demand rate and infinite production rate without shortages. Sharma, *et al.* ^[23] gave an inventory model of two-parameter exponential distribution with finite production. Sharma, *et al.* - correlated parameters and variables in an inventory model of deteriorating items involving fuzzy with shortages, exponential demand and infinite production rate.

Some commodities were observed to shrink with time by a proportion which can be approximated by a negative exponential function of time. The probability density function of a two-parameter exponential distribution is given by

$$f(t; \mu, \eta) = \frac{1}{\eta} e^{-\frac{(t-\mu)}{\eta}}, t \geq \mu, \eta > 1,$$

where μ is the location parameter and η is the scale parameter.
 The unreliability function is given by

$$F(t; \mu, \eta) = 1 - e^{-\frac{(t-\mu)}{\eta}}$$

The failure or hazard rate function of on-hand inventory is given by

$$H(t; \mu, \eta) = \frac{f(t; \mu, \eta)}{1-F(t; \mu, \eta)} = \frac{1}{\eta}, t \geq \mu, \eta > 1.$$

So the hazardous rate followed by the two-parameter exponential distribution is constant.
 In this paper, our objective is to analyse sensitivity of the parameters and variables. The results are summarized in tables.

2. Assumptions and Notations

1. d is the rate of demand, which is known.
2. $I(t)$ is the on hand inventory at any time t .
3. $H(t; \mu, \eta) = \frac{1}{\eta}$ is constant hazardous rate per unit time. It follows two-parameter exponential distribution ($0 < H < 1$).
4. C_p is the production cost of one item.
5. C_h is the inventory holding cost coefficient per unit time.
6. C_s is the shortage cost per unit time.
7. C_o is the operating cost per order.
8. C_d is the deterioration cost per unit time.
9. z is the order level to which the inventory raised at the beginning of cycle
10. s is the shortage at the end of time T
11. C is the average total cost per unit time.
12. T is the time of one cycle.
13. t_1 is the time of production.
14. I_1 is the maximum inventory level
15. Lead time is zero.
16. The inventory system deals with only one item.
17. $d, p, C_p, C_h, C_s, C_o, C_d, z, s, C, I, t_1, I_1 > 0; t \geq 0$.

3. Mathematical Model

The behavior of the inventory system is describing by the differential equations

The behavior of the inventory system is describing by the differential equations

$$\frac{dI(t)}{dt} = -d - \frac{1}{\eta} I(t) \text{ for } 0 \leq t \leq t_1 \text{ and } \frac{dI(t)}{dt} = -d \text{ for } t_1 \leq t \leq T \quad \dots \quad (1)$$

with boundary conditions $I(0) = z, I(t_1) = 0$ and $I(T) = -s. \quad \dots \quad (2)$

The solution of above system is obtained as

$$I(t) = \eta d \left(e^{\frac{t_1-t}{\eta}} - 1 \right) \text{ for } 0 \leq t \leq t_1 \text{ and } I(t) = -d(t - t_1) \text{ for } t_1 \leq t \leq T \quad \dots \quad (3)$$

with $z = \eta d \left(e^{\frac{t_1}{\eta}} - 1 \right)$ and $s = d(T - t_1) \quad \dots \quad (4)$

So the lot size per production run is $q = z + s = \eta d \left(e^{\frac{t_1}{\eta}} - 1 \right) + d(T - t_1) \quad \dots \quad (5)$

The situation of inventory is illustrated in the following figure:

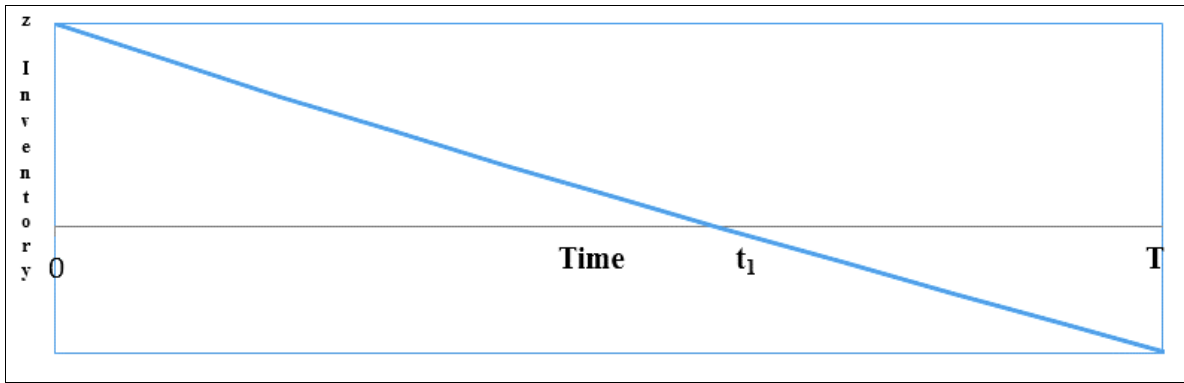


Fig 1

The total cost for one cycle includes the inventory holding cost, the shortage cost, the production cost and the ordering cost per cycle. So the total cost for one cycle of time T is

$$TC = HC + SC + PC + OC.$$

Hence the average total cost per unit time is given by

$$C = \frac{TC}{T} = \frac{1}{T} \left[\eta d C_h \left\{ \eta \left(e^{\frac{t_1}{\eta}} - 1 \right) - t_1 \right\} + \frac{1}{2} d C_s (T - t_1)^2 + C_p \left\{ \eta d \left(e^{\frac{t_1}{\eta}} - 1 \right) + d(T - t_1) \right\} + C_o \right]$$

Using $T = t_1 + t_2$, we have

$$C = \frac{1}{(t_1+t_2)} \left[\eta d \left\{ (\eta C_h + C_p) \left(e^{\frac{t_1}{\eta}} - 1 \right) - C_h t_1 \right\} + \frac{1}{2} d C_s (t_2)^2 + d C_p t_2 + C_o \right] = C(t_1, t_2)$$

For minimum value of C, Newton’s method is applied and hence we have obtained the optimum values t_1^* , t_2^* .

4. Numerical example

Taking the suitable values of parameters, the corresponding values of variables are obtained.

Table 1

C_h	T	C_p	T	C_s	T	H	T
3	1.325	16	0.9561	11	0.9621	0.006	0.8931
4	1.026	18	0.9263	13	0.9153	0.008	0.8903
5	0.9456	20	0.9248	15	0.9018	0.01	0.8896
6	0.8752	22	0.9215	17	0.8915	0.012	0.8815
7	0.8496	24	0.92	19	0.8768	0.014	0.8754
C_h	t_1	C_p	t_1	C_s	t_1	H	t_1
3	0.7652	16	0.6376	11	0.6456	0.006	0.6846
4	0.7321	18	0.6341	13	0.6541	0.008	0.6714
5	0.6548	20	0.6301	15	0.6661	0.01	0.6604
6	0.6376	22	0.6102	17	0.6792	0.012	0.6563
7	0.5931	24	0.6096	19	0.6863	0.014	0.6428
C_h	C	C_p	C	C_s	C	H	C
3	348.23	16	423.45	11	410.15	0.006	450.55
4	368.14	18	425.32	13	423.12	0.008	456.12
5	398.25	20	427.64	15	436.74	0.01	461.47
6	412.56	22	429.71	17	442.91	0.012	468.09
7	423.45	24	431.09	19	449.53	0.014	473.3

5. Conclusion

In this paper a sensitivity analysis is presented between parameters and variables. It is observed that the time of one cycle highly decreased by increasing holding cost, purchasing cost, shortage cost and hazardous rate; the inventory period highly decreased by increasing the holding cost, purchasing cost and hazardous rate but highly increased by increasing the shortage cost; the average total cost per unit time highly increased by increasing the holding cost, purchasing cost, shortage cost and hazardous rate. Hence we conclude that the average total cost increases highly with hazardous rate. Therefore it should be controlled for maximization of profit.

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7. References

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